A Learning Trajectory Framework for the Mathematics Common Core: Turnonccmath for Interpretation, Instructional Planning, and Collaboration

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Session Showcase:

Speakers:
- Alan Maloney, Ph. D., Senior Research Fellow, Mathematics Education, NC State University
- Jere Confrey, Ph.D., Chief Mathematics Officer, Wireless Generation & Joseph D. Moore Distinguished University Professor of Mathematics Education, North Carolina State University

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Theresa Gibson (NCSU), Project Manager
Shirley Varela (NCSU), Project Manager emerita
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Session Overview

- The CCSS-M context: oh them challenges
- How to make sense of the CCSS-M as a teacher: Why Learning Trajectories
- TurnOnCCMath Hexagon Map and Descriptors:
  - Unpack the Standards
  - Meld Learning Trajectories and the Common Core
  - Design, Structure, and Content
  - Examples from several LTs: LAV, VDM, RPP
- Teacher Responses to LTs and TOCC
- Implications for teachers/2b
- Resources to Come at TOCC
Then

- NCTM Standards (1989) and AAAS and NRC (Science)
- Standards state-by-state
- Beginning of NCLB
- Disaggregation of data
- Gaps in student achievement and opportunity to learn
- Nascent technology applications, graphing calculators
- Early internet, minimal access in schools
- Increasingly mobile student and teacher populations

Now

- Common Core State Standards, 45 states
- New designs for ESEA (eventually)
- Wireless networking and Cloud Computing (shared services model)
- Social Networking everywhere
- Data-intensive empirical research changing the way science is done, and different demands on quantitative modeling and literacy.
- Knowledge Intensive Industries; STEM
- Increasingly mobile student and teacher populations
- Opportunity by zip code; poverty and race-based gaps in opportunity to learn and student achievement
Recap of CCSS Development

- Very fast pace, and some topics ruled out ahead of time (stats and probability in K-5)
- July 2009: The development of the College and Career Ready Standards draft, outlining topic areas
- October 2009: Public release of the College and Career Ready Standards
- January 2010: Public release of Draft 1 to states
- March 2010: Public release of Draft 2
- June 2, 2010: Final release of Common Core State Standards with approval of the Validation Committee

(Note: These are NOT federal standards: they are a state-level coordinated effort led by National Governors Association-NGA and the Council of Chief State School Officers-CCSSO.)
As of June 2012, 45 states, D.C., and DoD Education Authority had adopted CCSS

http://www.corestandards.org/in-the-states
Criteria for developing the CC Standards

- Fewer, clearer, and higher standards
- Aligned with college and work expectations
- More rigorous content and application of knowledge through high-order skills
- Built upon strengths and lessons of current state standards, and...
- ...internationally benchmarked, with intent that all students prepared to succeed in “global economy” and society
- Intended to be adopted in common, across all states

CCSSI 2010; www.corestandards.org
CCSSM Congruence and NAEP Mathematics Performance:

*States with past mathematics standards that were more similar to the CCSSM had statistically significantly higher NAEP 2009 performance.*

W. Schmidt, 2012  
(MSU Center for the Study of Curriculum)

CCSS-M is now much more like a hypothetical “A+” set of international standards, with reduced variability state-to-state
Significance of the Common Core

- Expectations for higher level reasoning and skills
- Structured to incorporate student learning research and to support learning trajectories/progressions
- Economies of scale: coherence of standards across states, w.r.t. curriculum, assessment, PD
- Should buffer the effects of student mobility from state-to-state and resulting educational dislocation
- May turn out to support equity and customization
- (digital learning and use of real-time data)
This is NOT Business-as-Usual

1. Standards are more demanding; new and/or more intensified topic treatment, pacing

2. New assessments, new problem types to assess more complex reasoning

3. Continuing urgency to address student performance gaps (disequity of various kinds, student mobility)

4. Expanded data and new technologies transform instruction, career and college expectations
More demanding standards and new topics

A. Eight Mathematical Practices made explicit

B. Changes in Content and Grade Expectations (K-5, 6-8, 9-12): Earlier (or Later), and More Demanding

C. Interdisciplinary Content: Reading and Writing in Science and Technical Subjects
Eight Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

(CCSS-Math, 2010)
Mathematical Practices

• Practices must be interwoven with content. They must be addressed in tandem.

• Practices may provide a way to observe classrooms to see how the content is made understandable, challenging and engaging to students.
## Changes in Content and Grade Expectations

### K-5 Domains

<table>
<thead>
<tr>
<th>Domains</th>
<th>Grade Level</th>
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</thead>
<tbody>
<tr>
<td>Counting and Cardinality</td>
<td>K only</td>
</tr>
<tr>
<td>Operations and Algebraic Thinking</td>
<td>1-5</td>
</tr>
<tr>
<td>Number and Operations in Base Ten</td>
<td>1-5</td>
</tr>
<tr>
<td>Number and Operations--Fractions</td>
<td>3-5</td>
</tr>
<tr>
<td>Measurement and Data</td>
<td>1-5</td>
</tr>
<tr>
<td>Geometry</td>
<td>1-5</td>
</tr>
</tbody>
</table>
Changes by Grade Bands: grades K-5

- Numeralation and operation intensified, and introduced earlier
  - Early place value foundations in grade K
  - Regrouping as composing / decomposing, in grade 2
  - Decimals to hundredths in grade 4
- All three types of measurement *simultaneously*
  - Non-standard, English and Metric
- Emphasis on fractions as numbers
- Emphasis on number line as visualization / structure
## Changes: Middle Grades Domains

<table>
<thead>
<tr>
<th>Domains</th>
<th>Grade Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio and Proportional Relationships</td>
<td>6-7</td>
</tr>
<tr>
<td>The Number System</td>
<td>6-8</td>
</tr>
<tr>
<td>Expressions and Equations</td>
<td>6-8</td>
</tr>
<tr>
<td>Functions</td>
<td>8</td>
</tr>
<tr>
<td>Geometry</td>
<td>6-8</td>
</tr>
<tr>
<td>Statistics and Probability</td>
<td>6-8</td>
</tr>
</tbody>
</table>
Changes by Grade Bands: grades 6-8

- Ratio and Proportion focused on in grade 6, 7 (not earlier)
  - Ratio, unit rates, converting measurement, tables of values, graphing, missing value problems
- Percents introduced in grade 6
- Statistics introduced in grade 6
  - Statistical variability (measures of central tendency, distributions, interquartile range, mean and absolute deviation, data shape)
- Rational numbers in grade 7
- Grade 8: One-third of high-school algebra for all students
Changes by Grade Bands: grades 6-8

- Much higher expectations at middle grades,
  - 4th to 8th grade NAEP: where greatest performance drop-off occurs.
  - Collectively, our capacity, and our student performance, are weakest in middle grades
  - And no room/time to repeat topics each school year.

- Much less elementary preparation for major topics in middle grades (despite research): ratio, statistics, probability

- Much more pressure on elementary school teachers to “get the job done.”

- (Greater need to understand student learning, in order to accelerate it...)
Changes: High School Conceptual Categories and Domains

- **NUMBER AND QUANTITY**
  - The Real Number System
  - Quantities
  - The Complex Number System
  - Vector and Matrix Quantities

- **ALGEBRA**
  - Seeing Structure in Expressions
  - Arithmetic with Polynomials and Rational Expressions
  - Creating Equations
  - Reasoning with Equations and Inequalities

- **FUNCTIONS**
  - Interpreting Functions
  - Building Functions
  - Linear, Quadratic and Exponential Models
  - Trigonometric Functions
B. Changes: High School Conceptual Categories and Domains

- GEOMETRY
  - Congruence
  - Similarity, Right Triangles and Trigonometry
  - Circles
  - Expressing Geometric Properties with Equations
  - Geometric Measurement and Dimension
  - Modeling with Geometry

- STATISTICS AND PROBABILITY
  - Interpreting Categorical and Quantitative Data
  - Making Inferences and Justifying Conclusions
  - Conditional Probability and the Rules of Probability
  - Using Probability to Make Decisions

- MODELING
Changes by Grade Bands: grades 9-12

- One set of standards for all: all students and teachers are responsible for the same body of mathematical content and practices...
- ...CCSS-M does not dictate curriculum: could enact (a) a traditional siloed approach (Algebra I, Geometry, Algebra II) (b) an integrated approach, or (c) new models.
- All students must master some topics traditionally from algebra 2, or beyond
  - Simple periodic functions
  - Polynomials
  - Radicals
  - More probability and statistics (correlation, random processes)
  - Introduction to mathematical modeling
The transition to ‘full implementation’…

- Cross Walks?
  - Necessary: Tell you what is new or different,
  - But not sufficient: Need to know how areas and strands are restructured

- Standard by standard individual attention?
  - Necessary: To understand the standards, indicate central content
  - But not sufficient: Need to emphasize how concepts develop over time
Take-Away Messages, 1

- Many opportunities and challenges.
- New assessments will test more complex reasoning.
- Middle grades will be critical to student success.
- Elementary grades instruction critical to [--everything!—but most proximally] middle grades preparation and success.
- Implementation/transition should have considered phasing, sequencing, course naming, obligatory and ongoing professional development, and public relations.
- Specificity of standards should increase equity, if students are provided adequate opportunities to learn.
- Central hosting, technology services models, and diversity of content offers enormous opportunities and pitfalls.
Meaningful implementation of the CCSS-M: positive impact on student learning and achievement.

This is a generational project. And a legacy to young teachers.

But How to Make Sense of the CCSS-M?

- Continuity—conceptual, year-to-year
- Grain Size
- How to Interpret: Read? Visual Representation? In Class?
- Inservice teachers: transition? step up? retrofit?
- Preservice teachers: How to ensure they start careers assuming the CCSS-M (and what does this mean?)

An enormous undertaking
Learning Trajectories: what?

Connect (a) research on student learning of mathematics to (b) mathematics instruction.

- Mathematics learning is not synonymous with the structure of the discipline of mathematics.

Potential for improved instructional planning, anticipation of student strategies, representations, and misconceptions

CCSS-M built to incorporate learning progressions;

Anticipated revision based on further research on student learning:

“One promise of common state standards is that over time, they will allow research on learning progressions to inform and improve the design of Standards to a much greater extent than is possible today.”

CCSS 2010, p.5
Learning Trajectory within a Conceptual Corridor

Confrey (2006) Design Studies Chapter from the Cambridge Handbook of the Learning Sciences
Learning Trajectories--

...descriptions of children’s thinking and learning in a specific mathematical domain, and a related, conjectured route through a set of instructional tasks designed to engender those mental processes or actions hypothesized to move children through a developmental progression of levels of thinking, created with the intent of supporting children’s achievement of specific goals in that mathematical domain

(Clements & Sarama, 2004)
Learning trajectory/progression--

...a researcher-conjectured, empirically-supported description of the ordered network of constructs a student encounters through instruction (i.e. activities, tasks, tools, forms of interaction and methods of evaluation), in order to move from informal ideas, through successive refinements of representation, articulation, and reflection, towards increasingly complex concepts over time

(Confrey et al., 2009)
Underlying Features of Learning Trajectories

1. Emphasize big ideas that develop gradually over time

2. Describe the transition from *prior knowledge* to more sophisticated target understandings (*domain goal understanding*)

3. Identify intermediate understandings, and indicate how they can contribute to growth in conceptual understanding, and ways to recognize and build on these
Value of Learning Trajectories to Teachers

- Support understanding of student conceptual development and intermediate understandings
- Identify clusters of related concepts at grade level
- Unifying theory of student learning in the domain
- Suggest rich uses of classroom assessment
- Clarify what to expect about students’ preparation from previous year, and what will be expected of your students for next year.
- Support managing the range of preparation and needs of students—the more you understand about student learning of concepts and skills, the more readily you can identify tasks and discourse that supports improved proficiency
- Support cross-grade instructional collaboration and coordination
Instructional Core—
Learning Trajectories as Boundary Objects

Teacher’s Math Knowledge

Examination of Curricular Materials

Selection of Instructional Tasks

Fostering Discourse

Formative Assessment Practices

Diagnostic Assessment

Professional Development

Instructional Guidance and Improvement

Classroom Assessment

Monitoring

Formative Assessment Practices

Learning Trajectories

Standards

Intended Curriculum

Implemented Curriculum

Data

Data

(Confrey and Maloney, 2010)
CCSS-M Document Structure

• [1.] Grade

• [2.] **Domains**: larger groups of related standards. Standards from different domains may sometimes be closely related.

• [3.] **Clusters** of groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

• [4.] **Individual standards** that define what students should understand and be able to do.

CCSSO, 2010
Design and Organization

- **Content standards** define what students should understand and be able to do
- **Clusters** are groups of related standards
- **Domains** are larger groups that progress across grades
- **Fewer, clearer, higher...**

**Number and Operations in Base Ten**

Use place value understanding and properties of operations to perform multi-digit arithmetic.

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.
## Unpacking the CCSS-M: Learning Trajectories View, version 1 (charts)

<table>
<thead>
<tr>
<th>Place Value and Decimals</th>
<th>Kindergarten</th>
<th>Grade 1</th>
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</thead>
<tbody>
<tr>
<td>Content Strand</td>
<td>[K.NBT.1]</td>
<td>[1.NBT.2]</td>
</tr>
<tr>
<td><strong>Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., (18 = 10 + 8)); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.</strong> [K.NBT.1]</td>
<td></td>
<td><strong>Understand that the two-digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones - called a “ten.” b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).</strong> [1.NBT.2]</td>
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<td><strong>Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.</strong> [1.NBT.5]</td>
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<tr>
<td><strong>Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols &gt;, =, and &lt;.</strong> [1.NBT.3]</td>
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<tr>
<td><strong>Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</strong> [1.NBT.6]</td>
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**Quantity, Measurement, And Data**

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**Numeration, Operations, and Algebraic Thinking**

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**Geometry**

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To obtain posters, Order at [www.wirelessgeneration.com/posters](http://www.wirelessgeneration.com/posters).
Unpacking the CCSS-M

*Learning Trajectories to Interpret the Common Core: the Turn On Common Core Project*

[www.turnonccmath.net](http://www.turnonccmath.net)

Hexagon map of CCSS-M

Descriptors
Hexagon map of K-8
Common Core Math Standards
With Learning Trajectories Identified

© J. Confrey 2011
Hexagon map © Wireless Generation 2011
Turn On Common Core Math project

1. Goal: Interpret the CCSS-M from standpoint of student learning.

2. LTs embed the CC Standards, to articulate student learning developing over time (across grades)

3. Pragmatic: acknowledge CCSS-M importance; arrange to support understanding of student learning for instruction.

4. A resource for teachers, professional development, teacher educators, researchers.

5. Based on: research syntheses; experience revising NC Standards; hexagon representations of many state standards (collab. with WGen); multiple iterations of standards charts, service on CCSS-M validation committee.

6. *Not* curriculum
List of 18 K-8 Learning Trajectories

- Counting
- Place Value and Decimals
- Addition and Subtraction
- Equipartitioning
- Time and Money
- Length, Area and Volume
- Fractions
- Multiplication and Division
- Ratio and Proportion, and Percent
List of 18 K-8 Learning Trajectories

- Shapes and Angles
- Triangles and Transformations
- Elementary Data and Modeling
- Variation, Distribution and Modeling
- Chance and Probability
- Integers, Number lines, and Coordinate Planes
- Rational and Irrational Numbers
- Early Equations and Expressions
- Linear and Simultaneous Functions
Quick Tour--www.turnoncccmath.net

1. Two map views: grade level and LT
2. One standard per hexagon (to the #.AB.#.a level)
3. Hexagon: abbreviated text of standard, full-text visible
4. Descriptors:
   1. Structural Overview
   2. Full text of standard
   3. Extended discussion of standard’s content and implications for student learning
   4. Bridging standards
<table>
<thead>
<tr>
<th>CCSS-M Description</th>
<th>Descriptor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>K.MD.B</strong> Indirectly compare two objects by representing the attribute with, for example, another object and then directly comparing.</td>
<td>This Bridging Standard is introduced here to describe how students’ learning of measurements emerges. At the heart of the measurement learning trajectory is the movement from identifying attributes, to representing attributes, directly and indirectly comparing attributes, and finally unitizing attributes using constructed units and wisely choosing common units. [...]</td>
</tr>
</tbody>
</table>
Elements of LTs in Unpacking CCSS-M

www.turnoncccmath.net

1. Student strategies, representations (inscriptions), and misconceptions

2. Underlying cognitive and conceptual principles

3. Mathematical distinctions and multiple models

4. Coherent structure

5. Bridging standards
Variation, Distribution, and Modeling

- Describing the Distribution of a Set of Data
- Comparing Two Data Sets
- Sampling And Early Inference
- Bivariate Data, Scatter Plots and Basic Linear Regression

VDM LT developed from student learning research on statistical reasoning, including Lehrer and Schauble, Cobb, Lee and Hollebrands and Wilson, Shaughnessy, Konold, Garfield and Ben-Zvi, and others

<table>
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<tr>
<th>6</th>
<th>7</th>
<th>8</th>
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G R A D E S
Students engage in investigations that include the four steps in which they:

1. Pose a question
2. Collect relevant data
3. Analyze the data to answer the question
4. Interpret the results
Elements of Descriptors--VDM

1. Student strategies, inscriptions and representations, misconceptions
   - Ordered lists; case-value plots; ways to describe and measure variation
   - Pictographs, bar graphs, line (dot) plots, histograms, box plots
   - Misconception--variation implies lack of structure

2. Underlying cognitive principles
   - Distributions have shape, center, and spread
   - Central tendency involves varied notions of typicality, balance, middle

3. Emergent mathematical distinctions
   - Table of representations’ characteristics
   - The mean: as balance point, as fair share
   - Variation characterized: equipartitioned ordered set, mean absolute deviation

4. Coherent Structure
   - The process of conducting statistical investigations
   - Modeling--examination of what representations show and hide

5. Bridging standards
   - Adding circle graphs (K-5)
   - Peeling off box plots into its own section
Variation, Distribution, and Modeling

**Standard 6.SP.4 (Part 1)** Display numerical data in plots on a number line, including dot plots, histograms \[and box plots\].

*This standard occurs early in the LT, but “and box plots” is delayed until later in the trajectory…it is inappropriate to try to introduce all three representations, covering very different issues in variation, at the same time.*

**6.SP.4 (Part 2)** [Display numerical data in plots on a number line, including dot plots, histograms, and] box plot[s].
Complementary Approaches to Unpacking the CCSS-M

1. Student strategies, representations (inscriptions), and misconceptions

2. Underlying cognitive and conceptual principles

3. Mathematical distinctions and multiple models

4. Coherent structure

5. Bridging standards
Coherent Structure for Mathematical Reasoning: Length, Area, and Volume learning trajectory

LAV LT developed from student learning research on length, area, volume measurement (Clements and Sarama, Barrett, Battista, Lehrer, Nguyen and Confrey and colleagues)
Length, Area and Volume

Attributes

Length

Area and Perimeter

Volume

Conversions

Area and Volume of Geometric Shapes and Solids

<table>
<thead>
<tr>
<th>GRADES</th>
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<tbody>
<tr>
<td>K 1 2 3 4 5 6 7 8</td>
</tr>
</tbody>
</table>

NCCTM 2012 K. Lee and J. Confrey
These form a module that is done first for *length*, then *area*, then *volume*.
Additive principle

Compensatory principle

Measure: no gaps or overlaps

Indirectly compare

Directly compare

Conservation Principle

Define attributes

Length

Multiplicative principle

Compensatory principle

Measure: no gaps or overlaps

Indirectly compare

Directly compare

Conservation

Define attributes

Area, Volume
Complementary Approaches to Unpacking the CCSS-M

www.turnonccmath.net

1. Student strategies, representations, and misconceptions

2. Underlying cognitive and conceptual principles

3. Mathematical distinctions and multiple models

4. Coherent structure

5. Bridging standards
Ratio and Proportion, and Percents

Early Ratio Foundations

Percents

Ratio Boxes, Ratio Units, Unit Ratios, and Rates

Proportional Relationships

Unit Conversion

Graphing Proportional Relationships and Slope

RPP LT developed from student learning research on ratio, proportion, and percent, including the Rational Number Project (Behr, Harel, Post, Cramer, et al), Confrey, Hart, Noelting, Davis, Lamon, Streefland, Thompson, and others.

K-6 | 6 | 7 | 8

GRADES

11/19/12 - 11/26/12; Ratio and Proportion, and Percent
Ratio and Proportion, and Percents

Section 1: A New Form of Equivalence

Part 1: Using ratio tables

Part 2: Ratio Units and Unit Ratios

Part 3: Graphing

Part 4: Building Up

Part 5: Ratio Boxes
6.RP.1: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

6.RP.A: (Bridging Standard) Express the concept of a ratio using a table of values based on doubling, tripling, etc., and splitting using a variety of factors in both columns of the ratio table.
Standard 6.RP.B (Bridging Standard): Understand the concepts of ratio unit and unit ratio. Relate these concepts for a given table of values and show them on a graph.

- A ratio unit, or “base ratio,” is the smallest pair of whole numbers that defines the ratio.
- A unit ratio is a ratio in which one of the two values is equal to one.
Teacher responses: LTs and TurnOnCCMath

- Clements and Sarama LTs (early number, counting, shapes and attributes): very successful early grade curriculum and implementation


- Teachers incorporate conceptual framework and vocabulary to anticipate and interpret student work.

- Increasingly evidence-based, nuanced perceptions of students work

- Broader content knowledge (personal), and improved understanding of tasks’ relationships to equipartitioning concept
Teacher responses to LTs and TurnOnCCMath

- PD masters course, 2012
  - Add-on elementary math specialist certification program
  - Single-district cohort, 29 K-5 (6,7) teachers
  - Focus: Rational Number Reasoning. (EqPart, LAV, M/D, Fractions, and Ratio-Proportion-Percent LTs from TOCC)
  - Focus: Learning Trajectories and student learning

- LTs as framework for understanding student learning over time, vs. “logic of discipline”

- PD with respect to implementing the CCSS-M: Standards embedded within a framework of learning over time

- Link research and teacher journal literature to student learning and instructional practice
Teacher responses to LTs and TurnOnCCMath

- **Course design**
  - How LTs are developed (Clinical interviewing, experiments, critical importance of “student voice”) Research and Teacher journal literature
  - Support grappling with interplay of concept and procedure
  - Vertical teams for multi-week assignments
    - Read, analyze, critique, and propose instructional integration across grades, promote intergrade PLC
    - Lesson planning w.r.t. individual LTs, reinforce cross-grade coherence
Teacher responses to LTs and TurnOnCCMath

Outcomes

• Increasingly evidence-based, nuanced perceptions of students’ work, mathematical distinctions, argument
• Improved mathematical content knowledge(!)
• Grappling with progression of student learning, teachers reflect on and fill in gaps in their own understanding. LTs help them make sense of their own and their students’ intermediate knowledge, and growth paths.
• Makes sense of the (Common Core) Standards
• Hope for supporting student learning, even if saddled with a procedure-heavy curriculum

Challenges:

• Time and Grain Size of substantial PD efforts
  • Different approach to instruction and learning, urgency to implement CCSS, and not-business-as-usual
• Teachers as Learners: need iterations as they become more familiar.
Implications for Teacher Education Programs

TOCC Learning Trajectories: multi-dimensional framework for teacher learning and practice

• CCSS-M are a given, and are embedded, and represented in their entirety
• Teachers-to-be focus on the learner, on what and how the students are learning, growth in sophistication of reasoning, evidence for same
  • Anticipate specific student strategies, conceptions, progression of intermediate states
  • Anticipate their own moves, and notice their context and value
  • Grow instructional sensitivity and responsiveness more quickly
  • Multiplier effect?
• Reinforce classroom practices (Smith/Stein), high cognitive demand discourse (Hufferd-Ackles et al.)
• Teacher-as-learner: LTs as a vehicle for
  • Revisiting and revising their own learning of mathematics
  • Making sense of their own, as well as students’, intermediate or insecure understandings
  • Teaching as vehicle for continual learning and recursive enrichment, collaboration
• Improved math content knowledge: a not unintended consequence
TOCC Resources to come

Research literature citations for all 18 LTs.
Validation by external reviewers
Addressing the grain-size and dissemination dilemmas
  Presentations and workshops (top-level view)
  Powerpoints mapping LTs to the CCSS-M standards (intermediate level)
  MOOCs: fine-grained, broadly available, as-needed.
Links, spurs, and nodes: extended visualizations of connections among LTs.
Reflections and white papers
  So what about revising CCSS-M in light of new research?
What about LTs for the High School standards, eh?